10-3

(a) PW of costoptimistic = PW of benefitoptimistic

$80,000 = $20,000 (*P*/*A*, *i*%, 12)

$80,000/$20,000 = (*P*/*A*, *i*%, 12)

4 = (*P*/*A*, *i*%, 12)

Interpolation Method

Interest rate (*P*/*A*, *i*, 12)

20% 4.439

*a* *i* 4 *c*

*b* 25% 3.725 *d*

Since *a/b = c/d*

*i* = 20% + *a*

= 20% + *b (c/d*)

= 20% + 5% (4.439 − 4.000 )/(4.439 − 3.725)

= 20% + 5% 0.439/0.714

= 20% + 5% (0.6148)

= 20% + 3.07%

*i*optimistic = 23.07%

PW of cost most likely = PW of benefitmost likely

$80,000 = $20,000 (*P*/*A*, *i*%, 5)

$80,000/$20,000 = (*P*/*A*, *i*%, 5)

4 = (*P*/*A*, *i*%, 6)

Interpolation Method

Interest rate (*P*/*A*, *i*, 5)

7% 4.100

*i* 4

8% 3.993

*i* = 7% + 1% (4.100 – 4)/(4.100 – 3.993)

= 7% + 1% 0.100/0.107

= 7% + 1% (0.93457)

= 7% + 0.93457%

*i*most likely = **7.9345%**

PW of costpessimistic = PW of benefitpessimistic

$80,000 = $20,000 (*P*/*A*, *i*%, 4)

$80,000/$20,000 = (*P*/*A*, *i*%, 4)

4 = (*P*/*A*, *i*%, 4)

Interest rate (*P*/*A*, *i*, 4)

*i*pessimistic = 0.25%

(b) Mean life = (12 + 5 × 4 + 4)/6

= 36/6 = 6 years

PW of costmean life = PW of benefitmean life

$80,000 = $20,000 (*P*/*A*, *i*, 6)

$80,000/$20,000 = (*P*/*A*, *i*, 6)

4 = (*P*/*A*, *i*, 6)

Interpolation Method

Interest rate (*P*/*A*, *i*, 6)

12% 4.111

*i* 4

15% 3.784

*i* = 12% + 3% (4.111 − 4)/(4.111 − 3.784)

= 12% + 3% 0.111/0.327

= 12% + 3% (0.339)

= 12% + 1.018

*i* = 13.018%

Therefore, the estimated before-tax rate of return is 13.02%.

10-10

Probability of an accident or moving violation = 20% per year

Probability of paying additional $600 in the 0th year = 0

Probability of paying additional $600 in the 1st year = 20/100 = 0.2

Probability of accident only in the 1st year = (no accident in the 0th year) × (20% chance of accident in the 1st year) = 0.8 × 0.2 = 0.16

Probability of paying additional $600 in the 2nd year = (prob. of accident in the 1st year) + (prob. of accident in the 2nd year) = 0.2 + 0.16 = 0.36

Probability of accident only in the 2nd year = (no accident in 0th year) × (no accident in 1st year) × (accident in 2nd year) = 0.8 × 0.8 × 0.2 = 0.128

Probability of paying additional $600 in the 3rd year = (accident in 0th year) + (accident in 1st year) + (accident in 2nd year) = 0.2 + 0.16 + 0.128 = 0.488

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **Year 0** | **Year 1** | **Year 2** | **Year 3** |
| Probability Distribution | 0 | 0.2 | 0.36 | 0.488 |

10-12

(a)

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **First Cost** | **P** | **Net Revenue** | **P** | **Joint Probability** |
| $300,000 | 0.2 | $70,000 | 0.3 | 0.06 |
| 400,000 | 0.5 | 70,000 | 0.3 | 0.15 |
| 600,000 | 0.3 | 70,000 | 0.3 | 0.09 – pessimistic |
| 300,000 | 0.2 | 90,000 | 0.5 | 0.10 |
| 400,000 | 0.5 | 90,000 | 0.5 | 0.25 – most likely |
| 600,000 | 0.3 | 90,000 | 0.5 | 0.15 |
| 300,000 | 0.2 | 100,000 | 0.2 | 0.04 – optimistic |
| 400,000 | 0.5 | 100,000 | 0.2 | 0.10 |
| 600,000 | 0.3 | 100,000 | 0.2 | 0.06 |
|  | | | | **Total = 1.00** |

(b) optimistic: PW = –300,000 + 100,000 (*P*/*A*, 12%, 10)

= –300,000 + 100,000 (5.650) = $265,000

most likely: PW = –400,000 + 90,000 (5.650) = $108,500

pessimistic: PW = –600,000 + 70,000 (5.650) = –$204,500

10-20

Expected number of wins in 100 attempts = 100/38 = 2.6316

Results of a win = 35 × $5 + $5 bet return = $180.00

Expected winnings = $180.00 (2.6313) = $473.69

But since she must invest $500 in pursuit of these winnings, her expected loss is $500.00 − $473.69 = $26.31

10-28

|  |  |  |  |
| --- | --- | --- | --- |
|  | **Annual Savings** | **Probability** | **PW** |
| Optimistic | $80,000 | 20% | $213,440 |
| Most Likely | $60,000 | 50% | $85,080 |
| Pessimistic | $40,000 | 30% | –$43,280 |

PW values in the table come from the calculation:

PW = (Annual Savings)(*P*/*A*, 9%, 10) – $300,000

(a) E(Annual Savings) = $80,000(0.2) + $60,000(0.5) + $40,000(0.3) = $58,000

The PW using the expected values:

PW(Expected values) = $58,000 (*P*/*A*, 9%, 10) – $300,000

PW(Expected values) = $58,000 (6.418) – $300,000 = $72,244

(b) See tables for PW of individual estimates.

E(PW) = 0.2($213,440) + 0.5($85,080) + 0.3(–$43,280) = $72,224

(c) The answers match since the expected value for the annual savings is just a constant coefficient in the computation and we get a simple linear combination of the random variables. Compare to the results in Problem 10-48, where we employ a nonlinear factor in the calculations resulting in a disparity between the two answers.

10-36

From the data in Problem 10.7, the interest rate over the last 10 years has been 10%, 15% and 20%, with probabilities 0.2, 0.3 and 0.5 respectively. As shown in the solution to Problem 10.16, the expected value of the interest rate is therefore 16.5%. So the standard deviation of the interest rate is

s.d. = √EV2 – (EV)2

= √(6.52 × 0.2 + 1.52 × 0.3 + 3.52 × 0.5)

= √(15.25)

= 3.9%

10-38

E(PW) = 0.2 ($213,440) + 0.5 ($85,080) + 0.3 (–$43,280) = $72,224

Variance = ($213,440)2 (0.2) + ($85,080)2 (0.5) + (–$43,280)2 (0.3) – ($72,224)2

Variance = $$8,076,271,264

Standard Deviation = $89,868

10-40

Cost = $25,000

Interest = 12%

Life = 4 years

|  |  |
| --- | --- |
| **Annual Savings** | **Probability** |
| $7,000 | 0.3 |
| $8,500 | 0.4 |
| $9,500 | 0.3 |

Expected value = $7,000 (0.3) + $8,500 (0.4) + $9,500 (0.3)

= $2,100 + $3,400 + $2,850

= $8,350

EV2 = ($7,000)2 (0.3) + ($8,500)2 (0.4) + ($9,500)2 (0.3)

= $$14,700,000 + $$28,900,000 + $$27,075,000

= $$70,675,000

Riskstandard deviation = √EV2 – (EV)2

= √($$70,765,000 − $$69,722,500)

= √$$952,500

= $975.96

PWreturn = –$25,000 + $8,350 (*P*/*A*, 12%, 4)

= −$25,000 + $8,350 (3.037)

= −$25,000 + $25,358.95

= $358.95